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MODELLING CANADIAN SEARCH AND RESCUE OPERATIONS

BY

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and
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ABSTRACT

Issues of importance to Canadian search and rescue (SAR) operations are addressed in view of the forthcoming replacement of Canada's SAR helicopters. A goal programming model incorporating maximal coverage and *p*-median concepts in location theory is used to assess the level of service for the current SAR system. The operational performance of SAR aircraft is examined using simulations and classical queuing theory to establish best-, average-, and worst-case analyses of the expected steady-state time that an incident spends in the queue for both the current and the proposed future SAR system. The proposed system is shown to provide a significant improvement over the current one.

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MODELING CANADIAN SEARCH AND RESCUE OPERATIONS

INTRODUCTION

1. In the early 1990s, the Canadian Department of National Defence set the requirements for its new search and rescue (SAR) helicopter. The new helicopter was to have a minimum range of 500 nautical miles (nm) and a minimum cruising speed of 120 knots. In addition, it had to be able to recover from engine failure while in flight, hover at 6,000 feet above sea level, and fly in light icing conditions. After careful evaluation, the Cormorant - a variant of the EH-101 built by a consortium of British and Italian aerospace companies - was shown to meet all of the stated operational requirements.

2. The Cormorant helicopter program offers the additional advantage of an attractive industrial benefits package that includes the introduction of new ventures and the transfer of technology. The latter will provide Canada with an opportunity to establish a long-range, self-sustaining industrial base that will provide a competitive edge in the international marketplace.

3. In anticipation of the replacement of Canada's current SAR helicopters with the Cormorant, considerable effort has been expended to study the change in SAR performance that will likely result. This paper documents our approach in studying these issues.

4. The foremost concern of military planners is the level of service that the Cormorant will provide when operating out of the existing SAR bases: Will the current system provide adequate service and coverage of distress incidents, or will SAR bases need to be relocated to provide better coverage? What is the average service time (distance) between incidents that fall outside the provided coverage and the nearest SAR base? An answer to this second question is crucial if the operating costs are to be understood.

5. At first glance, it appears that the level of service for the optimal location of SAR bases could be determined easily by solving the classical "maximal set covering problem" (MSCP) (Daskin, 1995) to obtain the coverage of distress incidents. The optimal and current SAR systems could then be contrasted to determine effectiveness. The MSCP does not count incidents that lie beyond the maximum service distance as part of the SAR base's workload or responsibility, however. In view of this shortcoming, a goal-programming model incorporating maximal coverage and *p*-median concepts in location theory has been developed to assess the level of service for the current SAR system.

6. Another issue of importance is quantifying the operational performance of the Cormorant: How effective is the Cormorant compared to existing SAR aircraft? Does it shorten the expected steady-state time that an incident spends in the queue waiting for service?

7. In general, SAR operations are characterized by a non-homogeneous incident arrival pattern and non-Markovian service times. They belong to queuing systems of the *G/G/c* type and are mathematically intractable. As a result, simulations are used to compute the average-case analysis of the expected steady-state time that an incident spends in the queue. The validity of the simulation results is determined by constructing the upper and lower bounds (the best- and worst-case analyses) of the *G/G/c* queuing system using the Allen-Cunneen approximation (Allen, 1990). Valuable insights into the behavior of the system are also obtained using the theoretical bounds, such as the *maximum* time that an incident can expect to spend in the queue.

8. The organization of the paper is as follows: Section 1 provides some background information on SAR operations in Canada. Section 2 presents an analysis of the level of service provided by the SAR system. Section 3 reports on the development of a simulation model for comparing the operational effectiveness of the Cormorant to that of current SAR aircraft. This methodology is applied to analyze data obtained from the SAR base Comox, on the west coast of Canada.

1. BACKGROUND

9. According to the 1994 Defense White Paper, the Canadian Forces (CF) are responsible for air search and rescue operations. They are to provide significant resources to assist the Coast Guard in marine search and rescue, to assist local authorities in land search and rescue, and to operate the Rescue Coordination Centers which respond to thousands of distress signals every year. In general, search and rescue operations in Canada pose a significant challenge for CF personnel and their equipment. For example, the CF SAR helicopter must be able to endure Canada's harsh climate and must be able to work in Canada's sometimes rugged terrain and off all three of the nation's coasts. Safeguarding human life remains the top priority, and the CF will continue to play a major role in this area.

10. There are 33 CF facilities within Canada that can feasibly serve as SAR bases. These are displayed on the map in Figure 1. These locations include existing air bases with up-to-date maintenance facilities to service SAR rotary-wing and fixed-wing aircraft, as well as potential CF sites that can handle all SAR-related activities. At present, the SAR bases are located at Comox, Trenton, Greenwood, and Gander (represented by squares in Figure 1 and labeled as 1, 2, 3, and 4 respectively).

11. SAR incidents are classified into *air, marine, humanitarian,* and *civil,* and are categorized into *distress, potential distress* and *no danger.* This paper deals only with distress signals. Between 1992 and 1996, there were 34,573 SAR distress incidents reported; their locations are displayed in Figure 2.

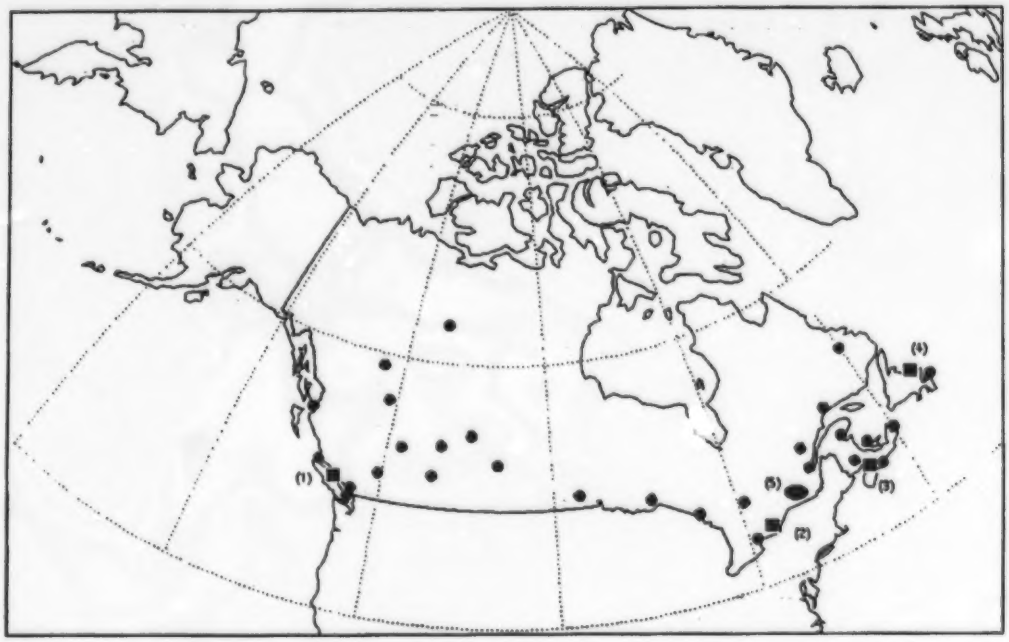


Figure 1 – Possible SAR Helicopter Base Locations

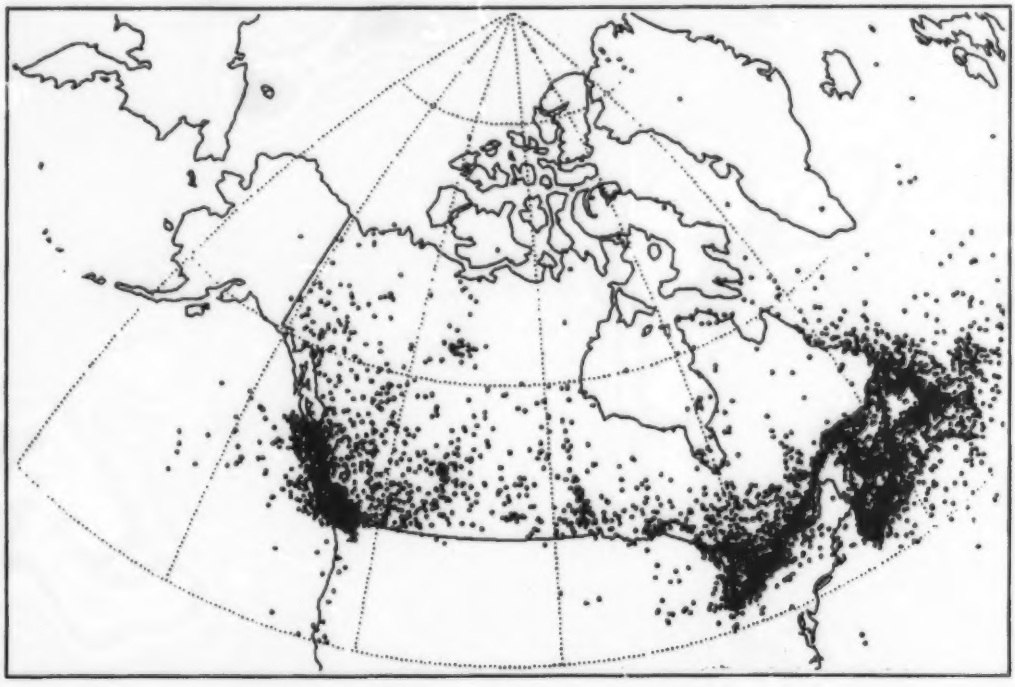


Figure 2 – SAR Distress Incidents Locations

12. Currently, Labrador helicopters and Buffalo fixed-wing aircraft are the workhorses of SAR operations in Comox; whereas Gander, Trenton, and Greenwood use Labrador and Hercules aircraft for search and rescue. The technical capabilities of these vehicles are summarized in Table I. The high maintenance costs associated with these aircraft, coupled with the aging of the fleet, signals the impending replacement of the Labrador helicopter for SAR operations.

13. The Cormorant is a civilian utility version of the EH-101. The engine to be fitted has not yet been chosen, but the competition is between General Electric's T700-GE-T6A engine and the Rolls-Royce RTM322. It is too soon to judge the fuel consumption that would result in each case. The operating characteristics of the Cormorant are summarized in Table I as well.

TABLE I
SAR AIRCRAFT TECHNICAL CAPABILITIES

	Labrador	Cormorant	Buffalo	Hercules
Range (nm)	500	530	1200	3300
SAR radius (nm)	200	200	500	1500
Speed (knots)	105	145	220	300
Fuel consumption rate (lbs/hr)	1100	1500-2000	2000	5000

2. LEVEL OF SERVICE

14. In this work, a preemptive goal-programming model is developed to optimally select the location of SAR bases. The primary concern for military planners is to locate SAR bases such that they guarantee service in as many distress incidents as possible within a coverage distance S; therefore, the first priority is to maximize the total number of incidents within radius S (i.e., to solve the MSCP). The next priority is to examine the average service time or distance between incidents that are outside the radius S and the nearest SAR base.

15. The Cormorant helicopter has the same SAR radius of operations as the Labrador that it is replacing (see Table I); therefore, the level of service offered by the Cormorant will be the same as that provided by the Labrador. These two helicopters have the smallest radius of operations among Canada's existing SAR aircraft. The coverage computed utilizing only the Labrador or Cormorant (S = 200 nm) will therefore establish

a minimum guaranteed level of service. Another reason why the Buffalo and Hercules are not used in the calculations is due to the age of the aircraft; they will eventually be phased out in the coming years.

16. Define

I = set of distress incidents;
J = set of 33 possible and feasible SAR facilities;
$Y_j = 1$ if SAR facility is located at site j, 0 otherwise;
$X_i = 1$ if distress incident i is within coverage distance S, 0 otherwise;
P = number of facilities to be located;
$Q = \{j \mid Y_j = 1\}$;
d_{ij} = distance from facility j to incident i;
$N_i = \{j \mid d_{ij} \le S\}$;
$D_i = \min_{j \in Q}(d_{ij})$;
$M = \{i \mid D_i > S\}$.

17. Thus, incidents $i \in M$ denote all those incidents that are outside the coverage of the closest SAR facility. The preemptive goal-programming model for the determination of the optimal SAR bases is given as follows:

$$\max\left(\sum_{i \in I} X_i\right) \tag{1}$$

$$\min\left(\sum_{i \in M} X_i \cdot D_i\right) \tag{2}$$

$$\sum_{j \in N_i} Y_j \ge X_i \qquad \text{for all } i \in I \tag{3}$$

$$\sum_{j \in J} Y_j = P \tag{4}$$

X_i, Y_j Binary (5)

18. Constraints (3) stipulate that incident i cannot be covered unless at least one of the sites that cover incident i is selected. Constraint (4) states that no more than P facilities can be located. Constraints (5) are the integrality constraints on the decision variables.

19. One approach to the solution of the above preemptive system is to decompose the system into sequence of two problems. First, a problem (I) is solved in which the objective function is given by (1) and the lower bound on the minimum total distance is dropped, subject to constraints (2-5). Then a second problem (II) - almost like the first - is solved, but the objective is replaced by (2) and an additional constraint is appended:

$$\sum_{i \in I} X_i \geq \text{optimal objective function value to problem (I)} \qquad (6)$$

20. Since X_i, Y_j are binary variables, the feasible region in the above system is not convex. As a result, there is no guarantee that the stated decomposition method will find the global solution. Maschler et al. (1979) discuss this idea in a game-theory setting. To compound the matter, the system (1-5) belongs to the class of *NP-hard* problems. That is, no efficient, provably optimal algorithm exists. For example, problem (II) is a variant of the classical *p-median problem.* When the number of facilities to be located is $p = 1$ (this case is called the *Fermat-Weber problem*), global optimality can be obtained by using an iterative descent method (Bazaraa et al., 1993), but as the number of SAR bases increases, the complexity of the problem rises dramatically. Cooper (1963) proved that for $p > 1$, the objective function of the p-median problem is neither convex nor concave and the combinatorial size of an exhaustive enumerative algorithm is extremely large (Stirling number of type II). Meggiddo (1984) proved that the p-median problem is NP-hard.

21. In light of these difficulties, the use of an heuristic or approximate algorithm is justified when solving problems (I) and (II) for near-optimal solutions (Pirkul and Schilling solved a similar problem in 1991).

22. The use of heuristics - such as the greedy heuristic, the interchange heuristic, etc. (Daskin, 1995) - is already popular among location-theory researchers. Moreover, recent advances in simulated annealing (SA) and evolution programming (Mavridou and Pardalos, 1997) have helped to propel the applications of heuristics and stochastic search algorithms to a new height. In our approach, problems (I) and (II) will be solved using simulated annealing.

23. In brief, the solution of large-integer or combinatorial optimization problems using SA is based on an analogy with the physical annealing of solids (Kirkpatrick et al., 1983; Eglese, 1990). As with all heuristic methods, however, the implementation of SA cannot guarantee the attainment of an optimal solution, although it may produce very good solutions in reasonable times. On the other hand, SA is a general procedure that can be used in a wide variety of combinatorial optimization problems; it is also relatively easy to program and use.

24. Our SA technique is a modified form of that used by Connolly (1990). He observed that the SA algorithm is more efficient when using sequentially generated neighboring solutions rather than randomly generated ones. In our approach, the vertex interchange heuristic is used to generate neighboring states. Applying the modified SA algorithm to the 1992-1996 SAR data, the optimal coverage provided by the Labrador helicopter and the total travel distance between incidents outside the Labrador coverage radius S and the nearest SAR base are evaluated.

25. Using the parameters $P = 4$, $S = 200$ nm, it is found that the optimal locations for SAR bases are at Comox, Bagotville, Greenwood, and Trenton. The only difference in location between the existing and optimal SAR bases is Gander (SAR site number 4 in Figure 1) versus Bagotville (site number 5 in Figure 1 as depicted by an ellipse). The levels of service given by the two sets of SAR bases are displayed in Table II. Even though the current SAR bases cover a smaller percentage of incidents than the optimal configuration, the sum of the total distances between the incidents outside the coverage radius S and the nearest existing SAR base is also smaller. (The larger sum of total distances between incidents outside S and the optimal SAR bases is a result of the preemptive priority structure of the goal-programming formulation. In solving problem (II), if the right-hand side of constraint (6) were to be relaxed to smaller values than the optimal objective function value in problem (I), then the sum of the total distances would decrease and eventually become smaller than that of the existing SAR bases.)

26. It is important to note that in addition to its responsibilities in search and rescue functions, Gander has been serving other maritime bases using SAR aircraft for support functions (a secondary role for the aircraft). The cost of relocating the maintenance facilities from Gander to Bagotville might therefore be very high. As a result, implementation has been postponed until a thorough analysis of the SAR base relocation

cost can be completed. The analysis in the next section assumes that the Cormorant helicopters will be operating out of these bases.

TABLE II
LEVELS OF SERVICE BETWEEN CURRENT AND OPTIMAL LOCATION OF SAR BASES FOR RADIUS OF OPERATIONS S = 200 NM

	Percentage of coverage of distress incidents	Total distance between incidents outside the coverage S and the nearest SAR bases (Knm)
Optimal location of SAR bases	75.8	3,785
Current SAR bases	72.1	3,492

3. OPERATIONAL PERFORMANCE OF SAR AIRCRAFT

27. Search and rescue operations are complex endeavors. They do not fit neatly into the classical mold of a physically stationary server whose prospective "customers" (incidents) arrive and queue up until they receive service. The arrival of incidents is perceived through some means of telecommunication, such as an "emergency locator transmitter," from various locations within the country. The only place at which a "queue" can be identified is at the Rescue Coordination Center, where records indicate the time, origin, and nature of a succession of requests for rescue. The SAR aircraft must then travel to the locations of these incidents to provide the rescue. Thus, search and rescue service of distress incidents belongs to the class of *spatially distributed queues* (Larson

28. Presently at Comox, all distress incidents lying within an annulus bounded by 200 and 500 nm (hereby denoted as Region 2) are serviced by Buffalo aircraft; all distress incidents within a 200 nm radius (denoted as Region 1) are serviced by the Labrador and occasionally by the Buffalo, if the latter is not already engaged in SAR activities in Region 2 (see Figure 3). Thus, the Buffalo aircraft can be perceived as a server alternating between incidents in Regions 1 and 2. It is a server with ancillary activities (Hall, 1989).

29. Currently, Comox participates in approximately 1/5 of all search and rescue missions within the 500 nm limit. The remaining SAR missions are handled by the Coast Guard and local authorities. The resources dedicated to search and rescue missions out of Comox include one Labrador and one Buffalo, with 200 nm and 500 nm radii of operations respectively. Depending on the choice of engines and the possibility of carrying an extra fuel tank, the radius of operations for the Cormorant ranges from 200 to 300 nm. (For simplicity, it is assumed to be 200 nm in this paper.)

30. The aim of this section is to compare and contrast the SAR operational performances of the Labrador and the Cormorant. Since the Cormorant is not yet in service, there is no SAR transit time data collected. For evaluation purposes, it will be assumed that the transit time of the Cormorant = the transit time of the Labrador * (speed of the Labrador/speed of the Cormorant). The search times for the Labrador and the Cormorant are assumed to be identical. An analysis of the SAR data collected between 1992 and 1996 leads to the following observations:

31. *Observation 1: The distress incident arrival pattern is a non-stationary (or non-homogeneous) Poisson process; that is, it varies over time.*

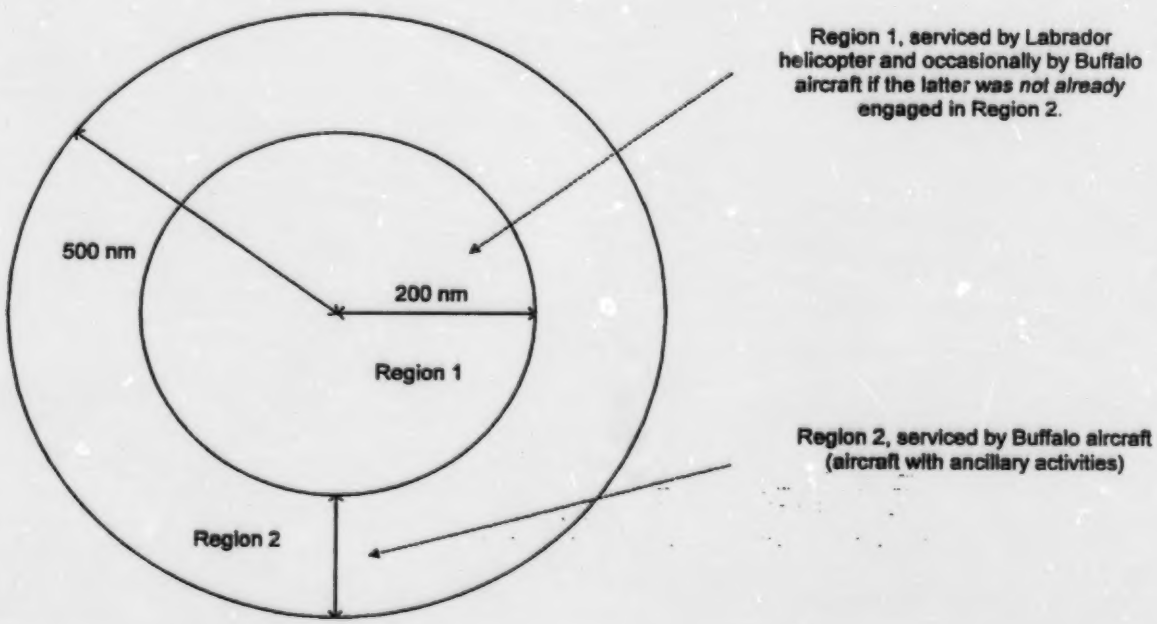

Region 1, serviced by Labrador helicopter and occasionally by Buffalo aircraft if the latter was not already engaged in Region 2.
500 nm
200 nm
Region 1
Region 2
Region 2, serviced by Buffalo aircraft (aircraft with ancillary activities)

Figure 3 – SAR Radii of Operations

31. First, in the SAR context, it is reasonable to assume that the probability that an incident arrives at any time does not depend on the times when other incidents arrive. Second, incidents are assumed to arrive one at a time. In addition, the incidents' arrival rate is not a constant; Figures 4a and 4b display the occurrence of incidents during a 24-hour period for both the summer and off-summer seasons. The occurrence of distress incidents is observed to be unsteady between seasons and more pronounced during the day, particularly in the afternoon, than at night. Finally, the erratic behavior of the incidents' arrival times, as shown by the graphs, renders further analysis (e.g., using statistical inferences) in demonstrating the non-stationary Poisson process property unnecessary. Note that the non-stationary Poisson process does not possess the property that interarrival times are exponential random variables.

32. *Observation 2: The random variable governing SAR service times does not possess the standard Markovian property. It cannot be represented by a negative exponential probability density function (PDF).*

Since

service time = transit time + search time

$$E(\text{service time}) = E\left(\frac{2}{v}\sqrt{D_x^2 + D_y^2}\right) + E(Z) = 1/\mu$$

Where

μ = service rate;

$E(.)$ = expected value of the random variable;

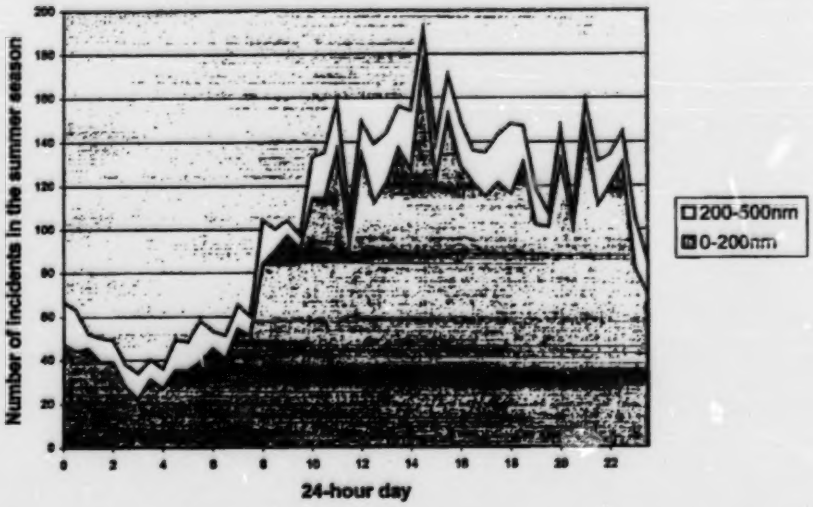


Figure 4a – Number of incidents as a function of the 24-hour day (summer season)

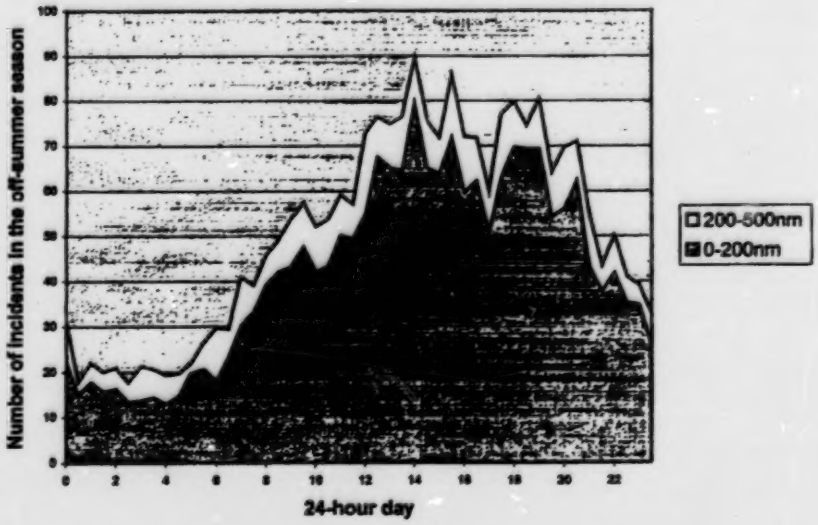


Figure 4b – Number of incidents as a function of the 24-hour day (off-summer season)

D_x, D_y = distances along the x and y axes respectively, from (to) the SAR base to (from) the location of incident;

v = speed of aircraft;

Z = random variable for search time.

33. Similarly, assuming that the search time of an incident is independent of the transit time, the variance for the service time can be obtained easily (Larson, 1981).

34. Define

$f_z(z)$ = probability density function for search time.

35. If $f_z(z)$ is assumed to be known (e.g., via curve fitting using the SAR database), then at least in principle an expression for the service time PDF can also be found (although it promises to be tedious and time consuming to compute). It suffices to note that the service time PDF cannot be a negative exponential unless $f_z(z)$ is a negative exponential and the expected time on the scene is much larger than the average round-trip time, in which case it can be argued that $1/\mu \approx E(Z)$ - and therefore the service time PDF - is approximately a negative exponential as well.

36. From the SAR database, it is evident that search times vary widely between incidents. All attempts to fit the service patterns statistically using a negative exponential PDF are in vain. In addition, one would intuitively expect that the SAR service time would rarely be exponential because of its memoryless property. One would also expect that the amount of time remaining in service would have to be dependent on the time already spent.

37. *Proposition* 1*: The G/G/c_1 queuing system, where c_1 = the number of Labrador helicopters, is distinctive of the physical process describing current SAR operations in Region* 1. *The G/G/c_2 queuing system with ancillary activities, where c_2 = the number of Buffalo aircraft, is distinctive of the physical process describing current SAR operations in Region* 2.

38. This proposition is obvious in view of the two observations. Note that it is well known that queuing systems of the *G/G/c* type are difficult to analyze mathematically. In addition, the queuing system $G/G/c_2$ describing the physical process in Region 2 has ancillary activities.

39. In subsection 3.1, a simulation model will be described. Simulation does not always provide as meaningful results as direct analysis, however. Upper and lower bounds utilizing the Allen-Cunneen approximation formula (Allen, 1990; Gross and Harris, 1998) for the *G/G/c* queue will be constructed in subsection 3.2. The bounds will furnish useful insights into SAR operations.

3.1 Simulation

40. In this work, an event-oriented simulation program has been developed in which the system is only evaluated at event times (arrival or departure time). The event-oriented simulation really amounts to two different simulations, one for arrival times and another for service times. The arrival times determine the arrival curve, while the arrival and service data are combined to determine the departure curves. Service times are simulated just like any other random variable: by generating $U[0,1]$ random variables and then inverting the empirical distribution function. The simulation of the non-stationary Poisson process is achieved using an activity-scanning procedure (Hall, 1991).

41. The simulation logic for dispatching SAR aircraft is represented by the flow chart displayed in Figure 5. Regions 1 and 2 are defined in Figure 3. Basically, Region 1 is the area within 200 nm of Comox and Region 2 is the annulus between 200 and 500 nm from Comox. Region 1 is serviced by rotary-wing aircraft (Labrador or Cormorant) and occasionally by Buffalo aircraft when the former are busy. Region 2 is serviced only by Buffalo aircraft.

42. Our program uses a terminating simulation approach with n independent replications (n seasons), where each of the n simulations is started with the same initial conditions. The initial conditions are set by letting the simulation run to allow for the warm-up or transient period to elapse prior to data collection. Furthermore, each replication is executed using a different sequence of random numbers; therefore, each replication can be treated as independent. Classical statistical analysis can then be used to

construct a 100(1-α)% confidence interval for the estimator of the measure of performance (Law and Kelton, 1982).

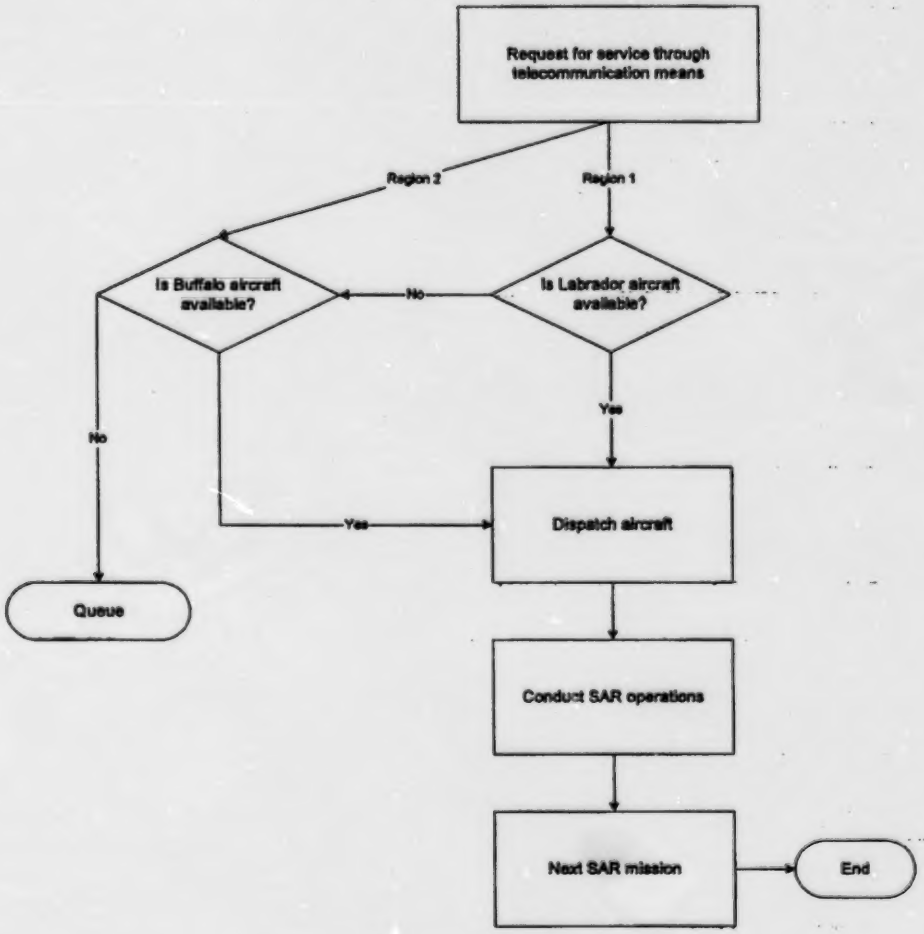


Figure 5 – Simulation of Search and Rescue Operations

43. Table III reports on the simulation results for W_q, the expected steady-state time that the incident spends in the queue. W_q is defined as the average waiting time, given by (time aircraft takes off or time aircraft is being re-diverted from potential distress/no danger incidents in order to service distress SAR incident) minus (time SAR incident is reported). As a reminder, distress SAR incident pre-empts potential distress and no danger incidents on the use of SAR aircraft an helicopters.

44. A 99% confidence interval for the mean waiting time is also included in the Table III (with a sample size of 1,000 seasons). The simulation results clearly indicate that the proposed SAR system (with the Cormorant replacing the Labrador) will reduce the expected time that the distress incident spends waiting in the queue by 47% for the summer season and by 44% for the off-summer season. The Cormorant thus greatly enhances the operational performance of the SAR system.

TABLE III
SIMULATION RESULTS ON W_q (IN MINUTES)*

	Summer season	Off-summer season
Current SAR system comprised of Labrador and Buffalo	8.04±0.27	3.66±0.26
Proposed SAR system comprised of Cormorant and Buffalo	4.26±0.18	2.58±0.17

*The interval denotes the 99% confidence interval for the mean waiting time.

3.2 Upper and lower bounds on W_q

45. The construction of the bounds relies on the approximation of the *G/G/c* queue using the Allen-Cunneen approximation formula (Allen, 1990).

Lemma: Allen-Cunneen Approximation Formula.

For any *G/G/c* queuing system,

$$W_q \approx \frac{C[c,a]\cdot W_s}{c\cdot(1-\rho)}\cdot\left(\frac{C_\tau^2 + C_s^2}{2}\right)$$

where

λ = mean arrival rate of incidents to the system;

μ = mean service rate per server;

τ = random variable describing interarrival time; $E(\tau) = 1/\lambda$;

s = random variable describing service time; $E(s) = 1/\mu$;

$a = \lambda/\mu$, $\rho = a/c$;

W_q = expected steady state time the incident spends in queue;

W_s = expected incident service time; $W_s = 1/\mu$;

$$C[c,a] = \text{Erlang's delay formula} = \frac{a^c / c!}{c \cdot (1-\rho) \cdot \sum_{n=0}^{c-1} \frac{a^n}{n!} + \frac{a^c}{c!}};$$

$$C_\tau^2 = \text{var}(\tau) / E(\tau)^2 \; ; \; C_s^2 = \text{var}(s) / E(s)^2 .$$

46. It is worth mentioning that the Allen-Cunneen Formula is exact for both the *M/M/c* and *M/G/*1 queues, and gives identical results when compared with other classical methods for solving the *M/G/c* queue (Allen, 1990).

47. *Proposition 2a: For current SAR operations in Regions* 1 *and* 2, *a worst-case scenario for the expected steady-state time that an incident spends in the queue is* $max\{W_{q,L}, W_{q,B}\}$, *where* $W_{q,L}$ = *expected time that the incident spends in the* $G/G/c_1$ *queue, with* c_1 *being the number of Labrador helicopters serving Region* 1; $W_{q,B}$ = *expected time that the incident spends in the* $G/G/c_2$ *queue (without ancillary activities), with* c_2 *being the number of Buffalo aircraft serving Region* 2. *That is, the Buffalo aircraft only serves incidents in Region* 2.

48. From the definition of $W_{q,B}$ and Proposition 1, it is evident that $max\{W_{q,L}, W_{q,B}\}$ constitutes the worst-case analysis of the expected time in the queue.

49. *Proposition 2b: In the proposed future SAR system, with the Cormorant helicopter replacing the Labrador, the corresponding worst-case scenario for the expected steady state-time in the queue is given by* $max\{W_{q,C}, W_{q,B}\}$, *where* $W_{q,C}$ = *the expected time that the incident spends in the* $G/G/c_3$ *queue, with* c_3 *being the number of Cormorant helicopters serving Region* 1 *and* $W_{q,B}$ *as defined in Proposition* 2a.

50. *Proposition 3: For current and proposed future SAR operations in Regions 1 and 2, a best-case scenario for the expected steady-state time that an incident spends in the queue is $W_{q,B}'$, where $W_{q,B}'$ = the expected time that the incident spends in the G/G/c_4 queue, with c_4 denoting two Buffalo aircraft.*

51. By virtue of the radii of operations for the Buffalo, Labrador, and Cormorant, the conclusion of this proposition is evident.

52. Using the above propositions and the Allen-Cunneen formula, $W_{q,L}$, $W_{q,B}$, $W_{q,B}'$ and $W_{q,C}$ are computed to obtain the upper and lower bounds on the expected steady-state time in the queue. These bounds, combined with the simulation results in Table III, are plotted in Figure 6 to highlight the worst-, average- and best-case analyses of the expected time in the queue. For example, in the summer season, given the current SAR system, it is expected that an incident will spend no more than 15.48 minutes waiting in the queue. On the other hand, an incident can expect to wait at least 1.56 minutes before any aircraft can possibly attend to it. Finally, one would expect the distress incident to spend, on average, approximately 8.04 mins waiting for service. Similar interpretations can easily be provided for the off-summer season and the proposed future SAR system.

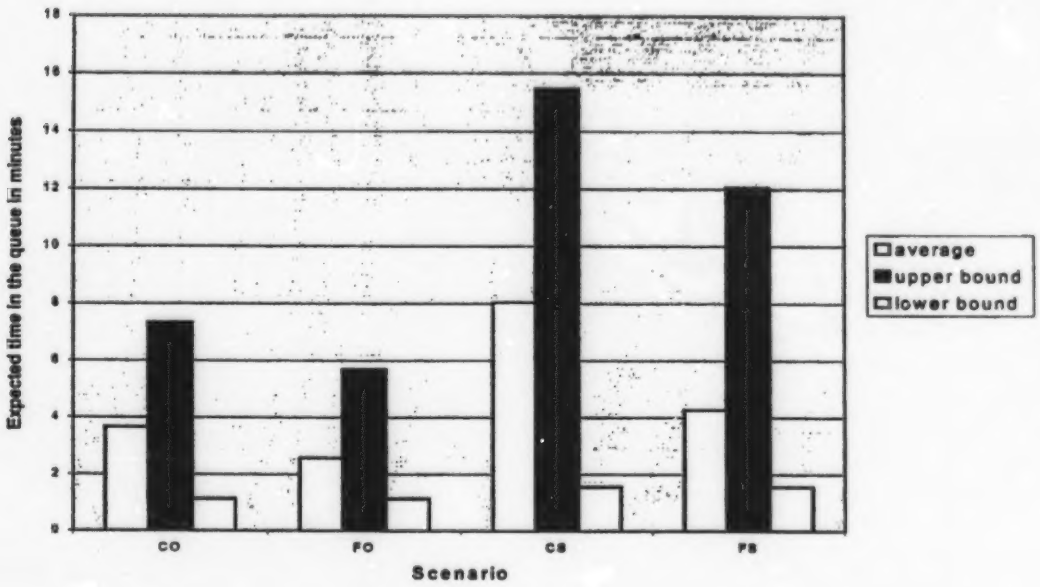


Figure 6 – Average, Best and Worst Case Analysis on the Expected Time an Incident Spent in the Queue
(CO: current system in off-summer season; FO: proposed future system in off-summer season; CS: current system in summer season; FS: proposed future system in summer season)

4. CONCLUSIONS

53. Some of the pertinent issues arising in SAR operations in light of the replacement of the Labrador helicopter by the Cormorant have been discussed. The important issue of the level of service provided by SAR bases has been addressed. A multi-objective programming model incorporating maximal coverage and *p*-median concepts has been developed to assess the level of service.

54. The above results were well received by military planners. Based on our analysis, military planners would also like to consider the option of adding an additional SAR base. Thus, implementation of the model has been postponed until a thorough comparative analysis of the cost of SAR base relocation versus the cost of adding an additional SAR base can be completed, however.

55. In addition, the simulation results and the queuing bounds found in this work quantify the operational performance of the new helicopter - the Cormorant - in Comox, on the west coast of Canada. On average, the proposed future SAR system in Comox (comprised of one Cormorant and one Buffalo) will reduce the expected steady-state time that an incident spends in the queue by 47% and 44% for the summer and off-summer seasons respectively. The corresponding maximum times that an incident can expect to spend in the queue are 12.1 mins and 5.7 mins respectively - a 22.1% improvement over the current SAR system (comprised of one Labrador and one Buffalo). These results confirm the simple analysis performed on the helicopters at the time of purchase, when the lack of complete SAR data and a tight deadline precluded a detailed analysis being conducted. Of most importance, this analysis has established a baseline for future performance evaluations

56. Finally, the methodology described in this paper will be generalized to examine the operational performance of current and future SAR system for all distress, potential distress and no danger SAR incidents. This analysis involves the study of queues with priorities for service and subject to interruptions. The findings will be reported in a subsequent paper.

REFERENCES

1. Allen, A.O. *Probability, Statistics, and queuing Theory with Computer Science Applications, 2nd edition* (Academic Press, 1990).

2. Bazaraa, M.S., H.D. Sherali and C.M. Shetty. *Nonlinear Programming, 2nd edition* (John Wiley & Sons Inc., 1993).

3. Connolly, D.T. "An improved scheme for the QAP," *European J. of Operational Research* **46** (1990), pp. 93-100.

4. Cooper, L. "Heuristic Methods for Location-Allocation Problems," *SIAM Review* **6** (1964), pp. 37-53.

5. Daskin, M. *Network and Discrete Location* (John Wiley & Sons Inc., 1995).

6. Eglese, R.W. "Simulated Annealing: A tool for Operational Research," *European J. of Operational Research* **46** (1990), pp. 271-281.

7. Gross, D. and C.M. Harris. *Fundamentals of Queuing Theory, 3rd edition* (John Wiley & Sons Inc., 1998).

8. Hall, R.W. "Expected Performance of a Queuing System with Ancillary Activities," *J. of the Operational Research Society* **40** (1989), pp. 741-750.

9. Hall, R.W. *Queuing Methods* (Prentice-Hall Inc., 1991).

10. Kirkpatrick, S., C.D. Gelatt, Jr. and M.P. Vecchi. "Optimization by Simulated Annealing," *Science* **220** (1983), pp. 671-680.

11. Larson, R.C. and A.R. Odoni. *Urban Operations Research* (Prentice-Hall Inc., 1981).

12. Law, A.M. and W.D. Kelton. *Simulation Modeling and Analysis* (McGraw-Hill Inc., 1982).

13. Maschler, M., B. Peleg and L.S. Shapley. "Geometric Properties of the Kernel, Nucleolus, and Related Solution Concepts," *Math of OR* 4 (Nov. 1979), pp. 303-338.

14. Mavridou, T.D. and P.M. Pardalos. "Simulated Annealing and Genetic Algorithms for the Facility Layout Problem: A Survey," *Computational Optimization and Applications* 7 (1997), pp. 111-126.

15. Meggiddo, N. and K.J. Supowit. "On the Complexity of Some Common Geometric Location Problems," *SIAM J. Computing* 13 (1984), pp. 182-196.

16. Pirkul, H. and D.A. Schilling. "The Maximal Covering Location Problem with Capacities on Total Workload," *Management Science* 32 (1991), pp. 233-248.

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Issues of importance to Canadian search and rescue (SAR) operations are addressed in view of the forthcoming replacement of Canada's SAR helicopters. A goal programming model incorporating maximal coverage and *p*-median concepts in location theory is used to assess the level of service for the current SAR system. The operational performance of SAR aircraft is examined using simulations and classical queuing theory to establish best-, average-, and worst-case analyses of the expected steady-state time that an incident spends in the queue for both the current and the proposed future SAR system. The proposed system is shown to provide a significant improvement over the current one.

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